SAN DIEGO MATH LEAGUE MIDDLE SCHOOL DIVISION, ROUND 2-B
January 2010

1. What is the value of $9999^2 - 1$?
   (A) 99999998 (B) 99980000 (C) 99990000 (D) 99989999 (E) None of them above.

2. Let the binary operation $\odot$ be defined by $a \odot b = (a+b)^2 - a^2 - b^2$. What is $10 \odot \frac{1}{2}$?
   (A) 10 (B) 11 (C) 12 (D) 13 (E) none of the above

3. In the problem below, each letter stands for a different digit.

$$\begin{array}{r} 1\ A\ B\ C\ D\ E \\ \times \qquad\qquad\quad 3 \\ \hline A\ B\ C\ D\ E\ 1 \end{array}$$

   What is A?
   (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

4. How many of the first 1000 positive integers are divisible simultaneously by $3, 4, 5$ and 6?
   (A) 12 (B) 13 (C) 14 (D) 15 (E) 16

5. Six numbers form an arithmetic sequence. The mean of the numbers equals the range. If the smallest number is 40, what is the biggest number?
   (A) 120 (B) 140 (C) 160 (D) 180 (E) None of the above

6. How many odd, perfect-square factors does the number $2^4 \times 3^6 \times 5^{10} \times 7^9$ have?
   (A) 18 (B) 20 (C) 120 (D) 180 (E) None of the above

7. If $f(x) = \frac{1}{1-x}$ what is $f(1-t)$?
   (A) t (B) $\frac{1}{t}$ (C) $1-t$ (D) $\frac{1}{1-t}$ (E) None of the above

8. Consider a sphere with radius of 3 cm cut by a plane 2 cm from the center of the sphere. What is the radius of the circular section in centimeters?
   (A)$\sqrt{3}$ (B) 2 (C) $\sqrt{5}$ (D) $\sqrt{6}$ (E) None of the above

9. Let $S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots + \frac{1}{199} - \frac{1}{200}$. Then which of the following is a correct estimate for S?

   (A) $\frac{1}{2} < S < 1$ (B) $1 < S < 2$ (C) $2 < S < 3$ (D) $3 < S < 4$ (E) None of the above

10. When rolling two standard six-sided dice, what is the probability of getting a sum not smaller than 10?

    (A) $\frac{1}{5}$ (B) $\frac{1}{6}$ (C) $\frac{1}{7}$ (D) $\frac{1}{8}$ (E) None of the above

11. What is

$$\frac{1}{2+\frac{1}{2+\frac{1}{2+\frac{1}{2+\cdots}}}}?$$

    (A) $\frac{\sqrt{5}-1}{2}$ (B) $\sqrt{2}-1$ (C) $\sqrt{3}-1$ (D) 1 (E) None of the above

12. Adding integers containing only the digit 8 (such as 88 and 888). What is the smallest number of integers containing only the digit 8 (such as 88 or 888) whose sum is 1000?

    (A) 125 (B) 25 (C) 5 (D) 3 (E) None of the above

13. Consider the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{11}$. How many ordered pairs of positive integers (x, y) are solutions to the equation?

    (A) 1 (B) 2 (C) 3 (D) 4 (E) More than 4

14. In a solar system that contains three planets and a sun, the planet A takes 12 months to complete its orbit, the planet B takes 20 months and the planet C takes 30 months. Assume that they are in a line L in the year 2010. Which year is the next (first) time when they all lie on the same straight line again?

    (A) 2015 (B) 2020 (C) 2030 (D) 2040 (E) None of the above

15. x, y and z are real numbers satisfying $xyz = 1$. What is the value of

$$\frac{x}{xy+x+1} + \frac{y}{yz+y+1} + \frac{z}{zx+z+1}$$

    (A) 1 (B) 2 (C) 3 (D) 4 (E) Can not be determined